SIXTH GRADE MATHEMATICS

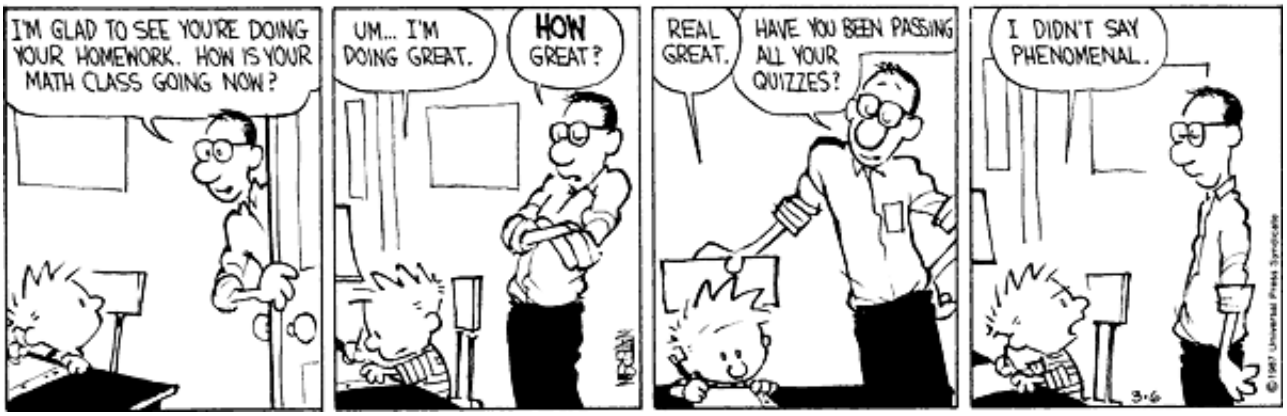
CHAPTER 3

NUMBER THEORY

TOPICS COVERED:

- Divisibility Rules
- Primes & Composites
- Prime Factorization
- Greatest Common Factor

What is an emirp number? It is a prime number that turns into another prime number when it is reversed. For example, 13 and 31 or 17 and 71. Did you notice that emirp is prime spelled backwards?



Created by Lance Mangham, 6th grade teacher, Carroll ISD

Teacher: Where is your homework? Student: I lost it fighting this kid who said you weren't the best teacher in the school.
If you plan for a year, plant a seed. If for ten years, plant a tree. If for a hundred years, teach the people. When you sow a seed once, you will reap a single harvest. When you teach the people, you will reap a hundred harvests. - Kuan Chung
Teacher: If 1+1=2 and 2+2=4, what is 4+4? Student: That's not fair! You answer the easy ones and leave us with the hard one!
The student came home from school with a long face. His dad asked, "What's wrong, son?" The boy replied, "The math test results came back today, Dad, and the teacher gave you a failing grade."
On arriving home from school, a little boy announced, "My math teacher is crazy." "Why?" his mother asked. "Yesterday," he said, "she told us five is four and one; today she is telling us that five is three plus two."
Little Tommy was in the first grade. One day, he came home and his mother asked: "Well, Tommy, what did you learn in school today?" "In math, I learned the three and three make seven." "But that's not correct," his mother said. "Well, then I guess I didn't learn anything."
Student: Wish I had been born 1,000 years ago! Teacher: Why is that? Student: Just think of all the history that I wouldn't have to learn!

Activity 3-2: **Divisibility Rules** Name:

Divisibility Rules

2 A number is divisible by 2 if the ones digit is even.	**3** A number is divisible by 3 if the sum of its digits is divisible by 3.
4 A number is divisible by 4 if its last two digits are divisible by 4.	**5** A number is divisible by 5 if it ends in 0 or 5.
6 A number is divisible by 6 if it is divisible by both 2 and 3.	**9** A number is divisible by 9 if the sum of its digits is divisible by 9.
10 A number is divisible by 10 if its last digit is 0.	

Circle the numbers that are divisible by 2.

34	58	67	90	241
324	243	432	423	234
196	825	4374	9701	65250

Circle the numbers that are divisible by 3.

48	75	76	77	78
761	762	763	764	765
46	51	913	834	7085

Circle the numbers that are divisible by 4.

934	924	944	954	964
732	742	752	762	772

Circle the numbers that are divisible by 5.

354	355	375	380	385
650	605	506	560	1056
325	608	5280	8542	49104

Circle the numbers that are divisible by 6.

78	62	3054	5553	24718
69300	762	765	96	104

Circle the numbers that are divisible by 9.

377	378	387	837	827
4876	5876	5976	9567	5796

Circle the numbers that are divisible by 10.

100	75	23	60	108
120	245	250	380	387

Activity 3-3: **Divisibility Rules** Name:

1.	The Southlake Carroll Marching Band is getting ready to perform at halftime of the football game. With 216 musicians, can the marching band form equal rows of 3? of 4? of 5? of 6? of 9?	
2.	True or false: All numbers divisible by 5 are also divisible by 10.	
3.	True or false: All numbers divisible by 10 are also divisible by 5.	
4.	True or false: All numbers divisible by 9 are also divisible by 3.	

Determine whether the first number is divisible by the second.

5.	185; 5		6.	76,870; 10		7.	461; 1	
8.	456; 3		9.	35,994; 2		10.	6,791; 3	
11.	12,866; 9		12.	7,564; 4		13.	45, 812; 9	

14.	A giant pizza is divided into 18 pieces. What are the different numbers of people you can divide it among so that there are no pieces left over?	
15.	You and 8 of your friends have been saving money by recycling aluminum cans. You have made a total of $58.86. Can you divide the money evenly among yourselves? How can you tell without dividing?	
16.	What is the smallest number you find that is divisible by 2, 3, 5, 6, 9, and 10?	

Complete the table. Answer *yes* or *no* for each box.

	Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 9	Divisible by 10
17.	324							
18.	475							
19.	525							
20.	600							
21.	1234							
22.	3951							
23.	4230							
24.	7803							
25.	9360							
26.	11,235							
27.	15,972							
28.	23,409							

29.	Marty said to Doc, “So we are going to travel back in time. What year did you set the Delorean for?” Doc replied, “I can’t remember exactly, but I do remember the following: If you divide the year by 2, you’ll get a remainder of 1. If you divide the year by 3, 4, 5, 6, 7, or 9, you’ll also get a reminder of 1.” “What about 8? Do you also get a reminder of 1?” “No,” said Doc. Marty then knew which year they were off to. Which year?

Activity 3-4: **Primes and Composites** Name:

1. Using the centimeter cubes, take one cube. Convince yourself that you make only one rectangle using one cube (remember a square is also a rectangle). Sketch the rectangle below. What are the dimensions of the rectangle?

2. Take two cubes. Determine how many unique rectangles (rotations are not unique) you can make with two cubes. Sketch them below.

3. What is a factor? What are the factors of 2? What is the relationship between the factors of 2 and the dimensions of the rectangles you made?

4. Continue in this manner with 3 cubes, 4 cubes, and so on. For each record in the following table the information requested.

Number of Tiles	# of Rectangles Made	Dimensions of each Rectangle	Factors	Prime?
1	1			
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

5. What is the relationship between the number of rectangles, the dimensions of the rectangles, and factors? Explain.

6. What is a prime number? Write a definition.

7. What is a composite number? Write a definition.

8. Determine which numbers are prime and which numbers are composite in your table on the previous page. Put a P or a C in the column.

9. Is 1 prime? Explain.

1. Circle all the prime numbers in the first row.

2. Draw a line through the first column (except for 2) and through the third and fifth columns.

3. Draw a line through the second column (except for 3).

4. Draw a diagonal line from the 5 in the top row (not including the 5) to the 5 in the side column. Repeat with diagonal lines between the pairs of 5's in the side columns.

5. Draw a diagonal line down and to the right between the first 7 in the left side column to the first 7 in the right side column. Do this again for the second 7's.

6. Circle any number that does not have a line through it.

7. Explain why you are left with just the primes.

Side Column	**First**	**Second**	**Third**	**Fourth**	**Fifth**	**Sixth**	**Side Column**
	2	3	4	5	6	7	
7A	8	9	10	11	12	13	
	14	15	16	17	18	19	5A
	20	21	22	23	24	25	
5	26	27	28	29	30	31	
	32	33	34	35	36	37	
	38	39	40	41	42	43	
	44	45	46	47	48	49	5B
7B	50	51	52	53	54	55	7A
5A	56	57	58	59	60	61	
	62	63	64	65	66	67	
	68	69	70	71	72	73	
	74	75	76	77	78	79	5C
	80	81	82	83	84	85	
5B	86	87	88	89	90	91	
	92	93	94	95	96	97	7B
	98	99	100	101	102	103	

5C

UCLA Mathematicians Discover 13-Million-Digit Prime Number

13 MILLION DIGIT PRIME NUMBER FOUND

Posted by ***RonInBayside*** *on* ***Sat Sep 27 15:02:44 2008***

UCLA mathematicians discover a 13-million-digit prime number
The mathematicians have found the first verified Mersenne prime number with more than 10 million digits, putting them in line to win a six-digit prize from the Electronic Frontier Foundation.
By Thomas H. Maugh II, Los Angeles Times Staff Writer , September 27, 2008

UCLA mathematicians appear to have won a $100,000 prize from the Electronic Frontier Foundation for discovering a 13-million-digit prime number that has long been sought by computer users.

While the prize money is nothing special, the bragging rights for discovering the 46th known Mersenne prime are huge.

"We're delighted," said UCLA's Edson Smith, leader of the effort. "Now we're looking for the next one, despite the odds," which are thought to be about one in 150,000 that any number tested will be a Mersenne prime.

Prime numbers are those, like three, seven and 11, that are divisible only by themselves and one. Mersenne primes, named after the 17th century French mathematician Marin Mersenne, who discovered them, take the form 2P - 1, where P is also a prime number.

In the new UCLA prime, P = 43,112,609.

Thousands of people around the world have been participating in the Great Internet Mersenne Prime Search, or GIMPS, in which underused computing power is harnessed to perform the complex and tedious calculations needed to find and verify Mersenne primes. The prize is being offered for finding the first Mersenne prime with more than 10 million digits.

Smith and his UCLA colleagues have, since last fall, harnessed the power of the 75 machines in the university's Program in Computing/Math Computer Lab, which is used by students for computer projects. Smith, a system administrator, realized that the lab was using only a fraction of its available CPU power. Rather than let it go to waste, he and his colleagues decided to use it for the GIMPS project.

The new Mersenne prime was discovered Aug. 23 on a Dell Optiplex 745 running Windows XP. The number was verified by a different computer system running a different algorithm.

The new prime is the eighth Mersenne prime discovered at UCLA. In 1952, mathematician Raphael Robinson found five of them using UCLA's Standards Western Automatic Computer. They were the 13th through 17th Mersenne primes discovered, the first ones found in more than 75 years, and the first to be discovered using a digital computer. Each had a few hundred digits.

In 1961, mathematician Alexander Hurwitz discovered two more, each with more than 1,200 digits, on the university's IBM 7090 mainframe.

Activity 3-7: **Primes & Composites** Name:

Erathosthenes, an ancient Greek mathematician, developed a method to determine prime numbers. His method for finding the 25 prime numbers between 1 and 100 is explained below. A **prime number** is a number with only 2 factors. **Composite numbers** have more than 2 factors.

1. Cross out 1. One is not prime because it only has one factor (1).
2. Circle the smallest prime number. What is it? _____ Cross out all multiples of this number.
3. Circle the next prime number. What is it? _____ Cross out all multiples of this number.
4. Circle the next prime number. What is it? _____ Cross out all multiples of this number.
5. Circle the next prime number. What is it? _____ Cross out all multiples of this number.
6. Circle all of the prime numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Using your 100 Board, answer the following questions.

1. What is the smallest prime number that is greater than 30?	
2. What is the smallest prime number that is greater than 50?	
3. 5 and 7 are called twin primes because they are both primes and they differ by two. List all twin primes between 1 and 100.	
4. Find 5 composite numbers in a row.	
5. Why didn't we have to keep going and circle all multiples of 9?	
6. Which of the primes 2, 3, 5, and 7 divide into 84?	
7. There are four columns on the board that contain no primes. Find them and explain why these columns contain no primes.	

Cross out the boxes containing composite numbers to discover the hidden message.

D 7	P 6	I 2	R 8	V 19	I 11	M 12	P 60	S 3	K 9	S 14	O 59	Z 35	R 11	S 37
Q 4	A 3	R 31	M 25	E 23	S 10	D 29	M 12	I 41	V 97	H 100	I 23	N 83	E 13	A 12
B 71	U 2	R 35	T 3	T 27	F 43	O 42	A 37	I 64	C 7	T 5	R 45	O 13	R 11	S 71
N 9	E 14	U 69	M 32	A 17	S 87	F 48	G 75	O 20	R 19	K 9	E 97	Q 8	T 27	D 57
F 67	R 2	C 16	I 89	M 18	E 7	T 12	K 9	N 17	D 73	L 67	N 49	I 59	E 29	R 83

Activity 3-8: **Primes Factorization** Name:

Determine whether each number is composite, prime, or neither.

1.	18		2.	31	
3.	1		4.	434	
5.	97		6.	111,111	
7.	57		8.	4,293	
9.	73		10.	38	

Create a factor tree to find the prime factorization of each number. Write your answer using exponents when necessary.

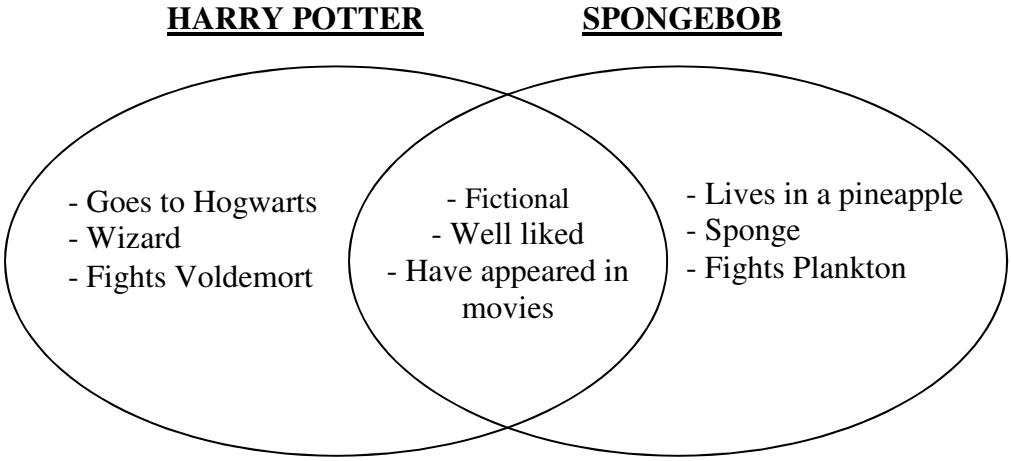
11.	280		12.	92	
13.	900		14.	20	
15.	35		16.	54	
17.	96		18.	64	
19.	30		20.	85	
21.	108		22.	45	
23.	88		24.	43	
25.	78		26.	84	
27.	1,024		28.	2,400	
29.	210		30.	128	
31.	324		32.	68	

Find the missing factor.

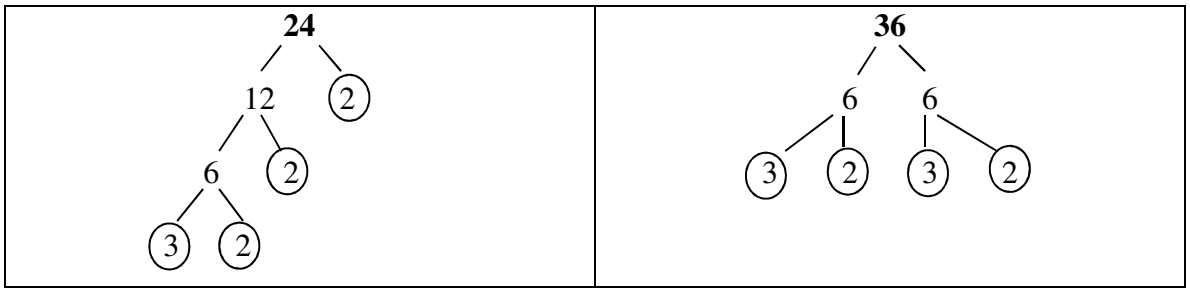
33.	$3^2 \bullet 5 \bullet$ ____ $= 315$	34.	$2^4 \bullet$ ____ $\bullet 7 = 1{,}008$
35.	$3^3 \bullet$ ____ $= 135$	36.	$2^2 \bullet 3^2 \bullet$ ____ $= 252$
37.	$5^2 \bullet$ ____ $= 275$	38.	$3^2 \bullet 5^2 \bullet$ ____ $= 2{,}475$

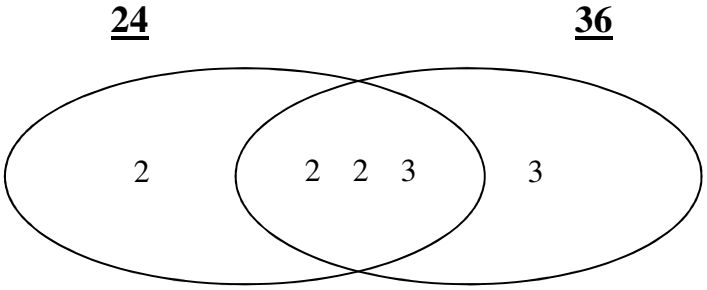
39.	What is the maximum number of prime numbered dates in any two consecutive months?	
40.	I am a two-digit prime number. The number formed by reversing my digits is also prime. My ones digit is 4 less than my tens digit. What number am I?	

Venn diagrams can be used to tell similar and different characteristics about two or more items. The Venn Diagram below compares and contrasts Harry Potter and SpongeBob.



Venn diagrams can be combined with a factor tree to find LCM's, GCF's, and for simplifying fractions.





GCF = multiply the factors that were in common = $2 \bullet 2 \bullet 3 = 12$

LCM = multiply all of the factors in the diagram = $2 \bullet 2 \bullet 2 \bullet 3 \bullet 3 = 72$

To simplify the fraction $\frac{24}{36}$ look at the factors outside the common area = $\frac{2}{3}$

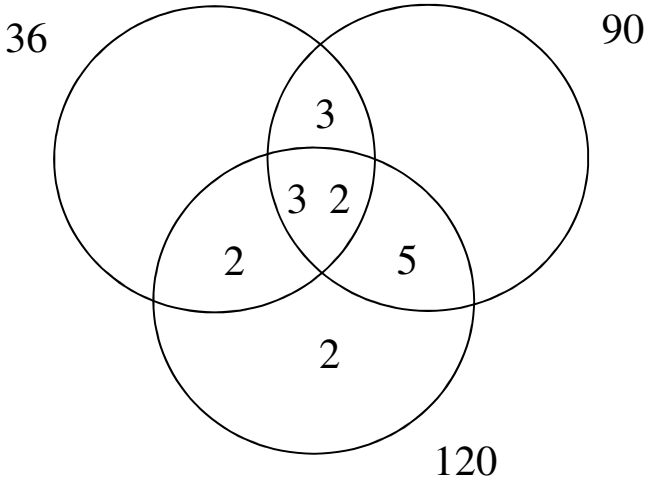
$24 = 2 \bullet 2 \bullet 2 \bullet 3 = 2^3 \bullet 3$ $\quad$ $36 = 2 \bullet 2 \bullet 3 \bullet 3 = 2^2 \bullet 3^2$

You can use a triple Venn diagram to find the GCF and LCM of set of monomials.

Below is an example of how to complete a triple Venn diagram.

Find the GCF and LCM of 36, 90, and 120.

The prime factorization of 36 = $3 \bullet 3 \bullet 2 \bullet 2$
The prime factorization of 90 = $5 \bullet 3 \bullet 3 \bullet 2$
The prime factorization of 120 = $5 \bullet 3 \bullet 2 \bullet 2 \bullet 2$



GCF = 6 because all three numbers have both a 2 and a 3 in common. 2 times 3 equals 6.
LCM = $5 \bullet 3 \bullet 3 \bullet 2 \bullet 2 \bullet 2 = 360$ because you multiply all terms in the diagram. Notice an easy way to do this is to take the 120 circle and just multiply it by the only number, 3, sitting outside this circle.

Activity 3-11: **Greatest Common Factor** Name:

The greatest of the common factors of two or more numbers is called the **greatest common factor** (GCF).

Finding the GCF by making a list

Find the GCF of 30 and 42 by making a list.

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

The common factors are 1, 2, 3, and 6. The greatest common factor is 6.

Find the GCF by making a list.

1.	12, 16		2.	18, 24	
3.	20, 16		4.	8, 4	
5.	30, 36		6.	35, 49	
7.	32, 40		8.	12, 18, 24	

Find the GCF of each set of using a factor tree and a tree diagram.

9.	14,28		10.	60,24	
11.	28,32		12.	48, 56	
13.	39, 26		14.	12, 20	
15.	36, 48		16.	40, 56	
17.	54, 72		18.	24, 30	
19.	25,30		20.	25, 27	
21.	20, 28, 36		22.	72,84,132	
23.	42,105,126		24.	14,28,42	
25.	33, 198, 330		26.	116,168,210	

27.	The GCF of two numbers is 850. Neither number is divisible by the other. What is the smallest that these two numbers could be?	
28.	The GCF of two numbers is 479. One number is even and the other number is odd. Neither number is divisible by the other. What is the smallest that these two numbers could be?	
29.	There were 91 hot dogs and 126 small cans of fruit juice for a math class picnic. Each student will receive the same amount of refreshments. What is the greatest number of students that can attend the picnic? How many cans of juice will each student receive? How many hot dogs?	

Activity 3-12: **Greatest Common Factor** Name:

Find the GCF by making a list.

1.	12, 13		2.	15, 25	
3.	15, 18		4.	27, 54, 81	
5.	16, 24 ,56		6.	21, 30, 44	

Find the GCF with factor trees and a Venn diagram.

7.	48, 72		8.	90, 150	
9.	88, 110		10.	85, 51	
11.	75, 90, 120		12.	45, 70, 120	

13.	A lady wants to buy plants for her garden and wants to plant them in rows with the same number of plants in each row. Which of these would give her the most choices for the number of rows she could make? Support your answer. 8 flats of 6 plants OR 7 flats of 8 plants OR 5 flats of 10 plants	
14.	What numbers between 50 and 100 have the greatest number of factors?	
15.	What calendar date(s) has the most factors? have exactly 3 factors?	
16.	Rebecca's little sister Tina has 48 yellow blocks and 40 green blocks. Tina builds some number of towers using all 88 blocks. What is the greatest number of identical towers that Tina can build? How many green and yellow blocks are in each tower?	
17.	Mr. Mangham has a certain number of cookies. They can be divided evenly among 9 students. They can also be divided evenly among 6 students. What are two possibilities for the number of cookies?	
18.	A band of pirates divided 185 pieces of silver and 148 gold coins. Since pirates are known to be fair about sharing equally, how many pirates were there?	
19.	Identify the number that satisfies all three conditions: a) It is a composite between 62 and 72. b) The sum of the digits is a prime number. c) It has more than 4 factors.	
20.	The number 6 has exactly four whole number divisors: 1, 2, 3, and 6. What is the smallest number with exactly five whole number divisors?	

Juniper Green – Round 1

Rules of the game:
1. Two players play at a time. The first player selects an even number.
2. On each turn, a player selects any remaining number that is a factor or a multiple of the number just selected by his or her opponent.
3. The first player who cannot select a number loses.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Juniper Green – Round 2

Rules of the game: The rules this time are very similar to the first game, except you and your partner are now working together. Try to stay alive as long as possible by crossing out as many numbers as possible. The game is over when a player cannot select another number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Activity 3-14: **The World's Easiest Quiz?** Name:

1.	How long did the Hundred Years War last?	
2.	Which country makes Panama hats?	
3.	From which animal do we get catgut?	
4.	In which month do Russians celebrate the October Revolution?	
5.	What is a camel's hair brush made of?	
6.	The Canary Islands in the Pacific are named after what animal?	
7.	What was King George VI's first name?	
8.	What color is a purple finch?	
9.	What country do Chinese gooseberries come from?	
10.	How long did the Thirty Years War last?	
11.	The maker does not want it; the buyer does not use it; the user does not see it. What is it?	
12.	Before Mt. Everest was discovered, what was the highest mountain on Earth?	
13.	In what year did Christmas and New Year's fall in the same year?	
14.	How many times can you subtract 5 from 25?	
15.	Even if they are starving, natives living in the Artic will never eat a penguin's egg. Why not?	